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A linear programming based decision support aid for Navy enlisted strength planning

Rodgers, Philip D.

Monterey, California. Naval Postgraduate School

NAVAL POSTGRADUATE SCHOOL

Monterey, California

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A LINEAR PROGRAMMING BASED
DECISION SUPPORT AID FOR NAVY
ENLISTED STRENGTH PLANNING

by

Philip D. Rodgers

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Thesis Advisor: Richard E. Rosenthal

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A Linear Programming Based Decision Support
Aid For Navy Enlisted Strength Planning

by

Philip D. Rodgers
Lieutenant, United States Navy
B.S., University of Illinois, 1982

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Author: [redacted]
Philip D. Rodgers

Approved by: [redacted]
Richard E. Rosenthal, Thesis Advisor

[redacted]
Robert F. Dell, Second Reader

[redacted]
Peter Purdue, Chairman
Department of Operations Research

ABSTRACT

A multi-objective linear program (MOLP) using goal programming is developed as a decision support aid in determining optimal levels of those areas of Navy enlisted strength planning which are subject to centralized management control. Over a multi-year period these decisions include monthly inventories in each paygrade, monthly total inventories, monthly advancements in the top six paygrades, and monthly recruiting goals. The model incorporates the various budgetary, Congressional, and internal Navy force structure constraints inherent in the strength planning process while minimizing deviations from desired inventory goals, ensuring inventory stability, and determining optimal recruiting goals.

THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

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I. INTRODUCTION

A. BACKGROUND

The U.S Navy, like most large military organizations, can be characterized as a nearly-closed, base-entry manpower system. In a system such as this, personnel enter mostly at the lowest paygrade and, over time, are subsequently advanced to higher grades or leave the service. Figure 1 shows a conceptual diagram of the force structure and the flows of personnel.

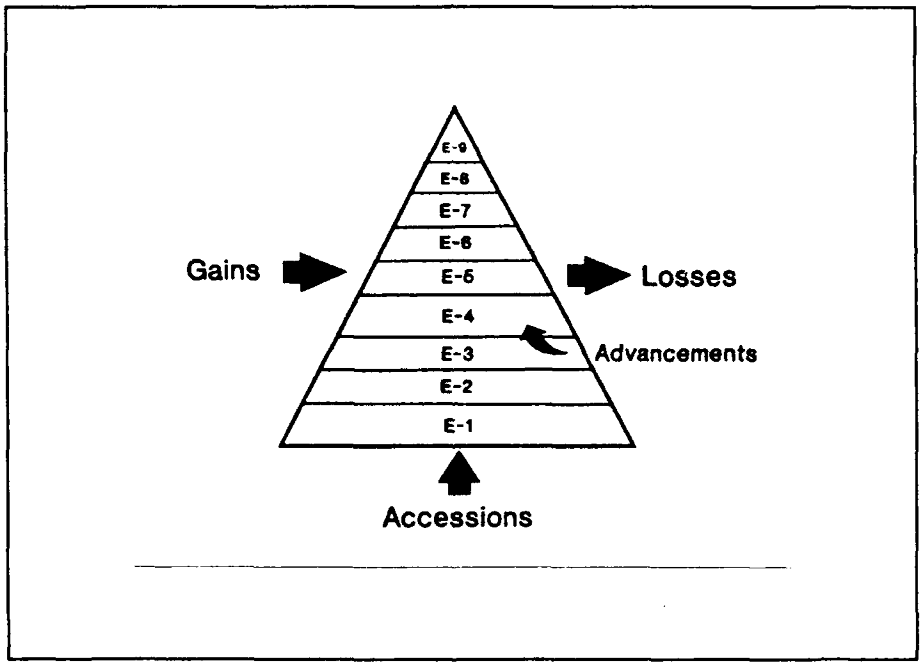

Figure 1. Stylized Enlisted Force Structure with Personnel Flows.

In recent years, the size of the Navy's enlisted force has been approximately 500,000 personnel with an annual budget to maintain that force of over $12 billion. It is the task of Navy personnel planners to manage the size and shape of this large inventory of personnel to obtain the most capable force possible while remaining within established budgetary and force structure limitations. In its most highly aggregated form, this process is referred to as "strength planning" where the term strength denotes the total number of personnel on hand. Due to significant differences in the policies and procedures used to manage the enlisted force versus the officer force, the Navy has established distinct and separate strength planners for each of the two populations. The model presented in this thesis is concerned with enlisted strength planning only, but the concepts in the model apply well to either population.

B. THE EXISTING ENLISTED STRENGTH PLANNING MODEL: SPAN

Currently, the Navy employs a manpower model entitled SPAN as its principle enlisted strength planning tool [Ref. 1]. Developed over twenty years ago by the Navy Personnel Research and Development Center, it is essentially a deterministic simulation model in which the planner is required to determine and input to the model all the values of the various decision variables. SPAN then generates a printed output which the planner must analyze in great detail to determine conformity with established goals and criteria. SPAN is a critical component of the overall Navy personnel management system in that data generated by SPAN is incorporated into a number of other personnel planning models within the Navy.

C. DEFICIENCIES OF THE CURRENT PROCESS

SPAN as it exists today has a number of shortcomings that cause it to be a very difficult and cumbersome model to use. As previously stated, the model requires a large amount of data to be input by the user. This data is ultimately put into a tabular format with some totals and averages computed. The model serves as little more than a report generator in this respect, and the output data must then be analyzed through some external means by the strength planner. This typically involves manually transcribing large amounts of data to various spreadsheets and graphs, along with a number of hand calculations to determine if the planned inventory meets all of the goals of the strength planner. This is a lengthy and time consuming process to simply perform one iteration of the modeling process. If the planned inventory fails to meet the objectives of the planner, they must then input revised data into the model and repeat the analysis process anew.

Another difficulty for users of SPAN is the complete lack of documentation for the model except for a disorganized collection of notes that various users have compiled over the twenty year lifetime of the model. With a new strength planner taking over the job every two to three years with little or no training from the preceding planner, the strength planning process has degenerated into an extremely inefficient and imprecise method to determine the future inventory needs of the Navy.

Finally, SPAN does not incorporate any optimization techniques in the formal sense to meet the many objectives and constraints faced by the strength planner. The process currently in place is a form of iterative improvement through trial and error since the model is run repeatedly to produce what appears to the planner to be an "optimal force". Clearly this method is extremely limited due to the time available to the planner to develop a feasible strength plan and the complexity of the interactions among the various decision variables. There is, of course, no guarantee of optimality.

It is worth noting that there are very few, if any, personnel planning models in operation within the Navy that incorporate optimization [Ref. 2]. No clear explanation exists as to why this is the case, however, it may be due in part to the fact that users of such models may find solutions derived from the "black box" of an optimization algorithm difficult to explain or modify. In addition, the existing approach of trial and error allows the user to account for constraints or objectives that are not explicitly included within the model. Most personnel planners in the Navy also have little or no training in the use and application of optimization techniques which may lead to an inherent mistrust of optimization models. One cannot be expected to place much faith in a model that is not well understood.

D. GOALS OF THE NEW SYSTEM

The goal of the new model (hereafter termed "LPSPAN" for linear programming counterpart of SPAN) described in this thesis was to develop a decision aid to assist the Navy's enlisted strength planner in determining optimal values of the various decision variables over which the Navy exercises centralized management control. These decision variables include monthly inventories by paygrade, monthly total strengths, monthly advancements in the top six paygrades, and monthly recruiting goals. In addition, the model was designed to incorporate budget and personnel costing factors not currently available in the existing strength planning model.

LPSPAN was also designed to free the strength planner from the repetitive and tedious task of running multiple iterations of the strength plan in an effort to obtain a plan which met all of the constraints imposed on the planner. Toward this end linear programming offered the most promise in providing fast, reliable decisions on how to manage the personnel inventory of the Navy. Underlying this decision was the objective of developing a rather straightforward application of optimization for personnel planning in an attempt to clearly illustrate the process and open up avenues for further applications within this very large part of the Navy's management structure.

A highly desirable feature for LPSPAN is the ability to run in a reasonable amount of time on personal computers currently available to the enlisted strength planner. A minimum of input data should be required for routine operation of the model, and the output of the model should clearly and succinctly display pertinent information to the user.

The model is intended to serve principally as a planning tool to determine optimal management criteria for future budget years rather than as a tool to be used during the execution of the current budget year inventory, although it is possible to operate the model for this purpose. In its role as a planning tool, LPSPAN should be flexible enough to deal with the many constraints that face the user in developing a strength plan, and should be able to clearly point out areas where potential problems might arise in meeting these constraints (e.g., insufficient budget resources are allocated to adequately man the Fleet).

E. OVERVIEW OF THE MODEL

1. Solution Approach

LPSPAN uses a widespread optimization technique known as "Goal Programming" to achieve its solution [Ref 3]. The optimal force in this case is defined as that force which provides the greatest military capability for a given budget cost. Military capability for purposes of the model is determined by the ability to meet the paygrade strength targets established by the Navy in the Enlisted Programmed

Authorizations (EPA). The EPA is an internal Navy planning document based on billet requirements throughout the Fleet. These requirements show the quantity and quality (by occupation and grade) of military personnel needed to accomplish assigned tasks and missions in wartime. The EPA ultimately serves as the "demand statement" to personnel planners who must strive to match the inventory of people as closely as possible to the requirements specified by the EPA.[Ref. 4]

LPSPAN employs a Markovian flow process to describe the behavior of personnel who are not subject to centralized control (e.g., gains and losses), a set of constraints which represent the institutional and budgetary considerations regulating the movement, size, and composition of the force at discrete monthly intervals over a multi-year planning horizon, and a set of penalty weights (which are incorporated as part of the objective function) for failing to meet any of the constraints. Optimal force planning requires selecting advancements, accessions, and inventories to minimize the weighted sum of the deviations from exactly satisfying any of the given constraints.[Ref. 5]

Since Navy enlisted strength planning involves satisfaction of a number of potentially conflicting objectives (e.g., achieving desired inventories while remaining within budget limits), LPSPAN can be categorized as a multi-objective linear program (MOLP). Goal programming allows the model to approach satisfaction of each goal without violating any of the constraints involved in the strength planning process. The user is afforded the opportunity to change the relative importance of achieving any single or combination of the multiple goals contained in the model.

2. Implementation

LPSPAN was written using the General Algebraic Modeling System (GAMS) [Ref. 6]. GAMS offers a number of attractive features for the development and continued use of a model such as LPSPAN.

- The language allows for very compact and easy to read code expressing the mathematical relationships within the model.
- Changes can be made quickly and easily as constraints or objectives change in the future.
- The model is completely portable from computer to computer with no machine specific changes required.
- Different solvers can be employed without changing the underlying model.

The model was developed and tested using actual strength planning and budget data from plans developed by the Navy for submission with the Presidential budget for fiscal years 1991 through 1993. The testing and verification of the model was performed on the Amdahl 5990 mainframe at the Naval Postgraduate School. LPSPAN has also been successfully run on several different personal computers with no change in the results obtained.

F. OVERVIEW OF RESULTS

LPSPAN has shown the ability to quickly and reliably produce results that are in line with existing strength plans and satisfy the constraints that exist today in the personnel strength planning environment. It produces larger inventories throughout the course of several fiscal years for the same cost as existing plans, with minimal turbulence in paygrade inventories, and consistently stable recruiting and advancements.

The particular instance of the model analyzed consists of 1405 equations with 2611 single variables making it a relatively small problem in comparison with other linear programming problems in operation throughout the military. An optimal solution has been found in 43 seconds using the MINOS 5.2 solver on the Amdahl 5990 mainframe and in 58 seconds on a 25 MHz 80486 personal computer using the CPLEX solver [Refs. 7,8].

II. MODEL FORMULATION

A. THE NECESSITY FOR MULTIOBJECTIVE OPTIMIZATION

Navy enlisted strength planners are concerned with a number of factors in developing a plan for managing the inventory of the Fleet. Principle among these is the desire to provide the Fleet with the largest possible inventory of personnel while remaining below Congressionally mandated ceilings. Secondly, it is imperative that the force be managed within strict budgetary guidelines. Along with the desire to maximize the total inventory of personnel is the need to produce a paygrade structure within the inventory that as closely as possible matches the paygrade structure requirements delineated in the EPA.

Good inventory management practices also dictate that stability be maintained in the force over time. Extreme fluctuations in the month to month inventory of any particular grade should be avoided. The Fleet must also have confidence that management practices will tend to reduce any variability in advancements over time. Although stability is not a requisite aspect of the strength planning process, it has significant ramifications in the day to day operations of the Fleet, as well as enhancing the credibility of the process and assisting in the maintenance of good morale.

In order to incorporate each of these many competing goals of the strength planner into a single unified mathematical representation, a multi-objective formulation with penalty weights assigned for failure to meet any single objective was chosen as an appropriate method [Ref. 9].

1. Budgetary Considerations.

As in any organization, the budget plays a pivotal role in the decision making process for the Navy. However, the Navy (as well as most Government organizations) has a somewhat different perspective toward management of the budget than is typically found in the private sector. While the Navy is allocated a fixed budget to be devoted to personnel, there is little incentive to "save" money by spending any less than the total amount appropriated. To do so could jeopardize future budget appropriations to the Navy by Congress.

Conversely, there is an exceptionally strong motivation to avoid exceeding the total budget that is appropriated. If this happens, it requires Congressional legislation to make up the difference, and would typically be made up through a reprogramming of budget resources from other Navy accounts (e.g., Operations and Maintenance). This is a politically arduous and unsavory process which in the past decade has forced the Navy in one instance to resort to a number of drastic management actions to avoid the need for reprogramming. The resulting personnel turmoil throughout the Fleet taught the Navy a difficult lesson in personnel management.

In view of these conflicting budgetary objectives LPSPAN uses goal programming to approach the exact budget figure (goal) as closely as possible. Penalty weights are assigned to both positive and negative deviations from this goal where the penalty assigned for exceeding the budget goal is substantially higher than the penalty assigned for falling short of the goal.

2. Congressional Force Structure Considerations

Congressional oversight of the military has led to several force structure considerations which must be adhered to in the strength planning process. Generally speaking, the constraints imposed on the enlisted forces of the various services are much less strict than those governing management of the officer community. Current law[1] limits the total size of the enlisted force in the Navy. The only other specifically delineated limitation to the composition of the enlisted force states that not more than one percent of the force may be in the E-9 paygrade, and not more than three percent in the E-8 and E-9 paygrades combined.[2] This constraint increases the potential for an infeasibility into the strength planning process since it often conflicts with the Navy's internal force structure requirements spelled out in the EPA. For example, the EPA for fiscal year 1991 calls for an E-8 and E-9 requirement of 15,454 personnel out of a force of 493,040, or 3.13%. These types of infeasibilities are common within the problem and make this an ideal application of goal programming in an effort to come as close as possible to the desired goals without violating any constraint.

3. Internal Navy Force Structure Considerations

For various reasons, the Navy establishes its own force structure goals in addition to the paygrade structure delineated in the EPA, which are not necessarily mandatory like the previous constraints, but are deemed desirable. The most typical example of this is establishing a desired percentage of the force to be in the top six enlisted paygrades (E-4 through E-9). The current tendency among the military

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[1]Public Law 101-510-Nov. 5, 1990, Sec. 401 (Fiscal Year 1991 Defense Authorizations Act)

[2]Title 10, U.S. Code, Sec. 517

services is a downward trend in the size of their forces and this is expected to be the case for the Navy at least through the mid 1990s before force sizes level off. As a tradeoff between smaller total force size and the need to continue operating the increasing number of high tech weapons systems entering the Fleet, the Navy has determined that the percentage of the force comprising the top six grades should go from 68.3% up to 70% during fiscal years 1991 through 1993. By increasing the percentage of personnel in the top six paygrades the Navy believes it will be able to maintain consistent advancement opportunities for their senior personnel in an environment of declining total inventories. These figures are certain to change in the future so the model has been designed to accommodate for future changes in Navy policy in this area.

B. INPUT PARAMETERS

LPSPAN has been formulated using three dimensions, or indices, on the various input parameters and decision variables.

- j = Enlisted Paygrade (E-1, E-2, ...,E-9)
- t = Month of Fiscal Year (Oct, Nov, ...Sep)
- y = Fiscal Year (FY1, FY2, FY3)

The particular instance of the model anlayzed considered a three year planning horizon, but the it could also be easily modified for larger problems involving longer periods of time.

The model was designed to limit the required input data to a reasonable amount. Most of the data should only need to be entered once per fiscal year or when updated information becomes available to the user. Several of these values should eventually become matters of policy and would not even change as the model is run over the course of many years. The following listing provides the nomenclature used by LPSPAN for the various parameters required as input. These parameters can be categorized as desired inventory levels, budgetary information, initial conditions, Markovian flow rates, variable bounds and structure constraints.

- $ENDFYEPA_{j,y}$ = Enlisted strength targets by grade for the end of each fiscal year. As published by the Navy, the EPA does not specifically establish requirements for E-2 inventories but covers this need in a large E-3 requirement. The model requires a specific value for E-2's so the E-3 figure was partitioned into E-2 and E-3 based on historical inventories in these grades. Monthly inventory targets are computed by a linear interpolation and stored internally to the model as $EPA_{t,j,y}$. Additionally, a trending factor is computed and stored internally for use in scaling the absolute deviation in month to month inventories as $SLOPE_{t,j,y}$.

- $COMPRATE_{j,y}$ = Annual "compensation rate" per individual utilized for determining the cost of the force. The figure includes a number of relevant costs associated with maintaining an individual in the force (e.g., retirement accrual, subsistence, FICA, in addition to base pay). Using this information the model computes monthly personnel costs and stores them as $MCOMP_{j,y}$.

- $BUDGET_y$ = Annual total budget for personnel expenditures.

- $BEGINSTR_j$ = Beginning inventory at the start of the problem.

- $BEGINEPA_j$ = Beginning EPA requirement at the start of the problem.

- $LR_{t,j}$ = Rate at which personnel in grade j leave the Navy during month t. These rates as well as the following three rates are incorporated into the Markovian flows of personnel within the force structure.

- $GR_{t,j\geq 2}$ = Rate at which personnel are gained from outside the Navy into grade j during month t. Note that there are no gains to E-1. Gains into the E-1 grade are treated as the decision variable RECRUITs.

- $AR_{t,j}$ = The rate at which personnel in grade j, month t are advanced to the next higher grade. Pertains to grades E-1 and E-2 only since advancements into E-2 and E-3 are decentralized and not subject to control by the strength planner.

- $DR_{t,j}$ = The rate at which personnel in grade j during month t are demoted to the next lower grade.

- $AUPFACT_j$ = Maximum percentage of grade j inventory that may be advanced in any month. This parameter as well as the following three parameters should be established by higher authority as matters of Navy policy rather than factors potentially manipulated by the user.

- $ALOFACT_j$ = Minimum percentage of grade j inventory that may be advanced in any month.

- $XUPFACT_j$ = Maximum fraction of grade j EPA requirement that grade j inventory can attain.

- $XLOFACT_j$ = Minimum fraction of grade j EPA requirement that grade j inventory can attain.

- $RECRUITUP_t$ = Maximum number of new recruits that can be brought into the Navy per month. This value represents the maximum monthly capacity of the Recruit Training Centers (RTCs) and is also linked to seasonal recruiting capabilities.

- $RECRUITLO_t$ = Minimum number of new recruits brought into the Navy per month. This value represents the minimum level at which the RTCs can operate without significant inefficiencies due to a lack of manpower.

- $T6_y$ = Desired percentage of personnel in the top six grades in year y.

- T1 = Maximum fraction of the force that can be in the E-9 grade.

- T2 = Maximum fraction of the force that can be in the E-8 and E-9 grades combined.

C. DECISION VARIABLES

The decision variables in LPSPAN have been chosen to reflect the kinds of "big picture" decisions faced by the strength planners in determining what actions are necessary and what the ultimate impact of those decisions will be. At the paygrade level of detail these inventory management decisions include:

- $X_{t,j,y} \geq 0$ Inventory of personnel in grade j during month t of year y.
- $A_{t,j,y} \geq 0$ Advancements made into grade j during month t of year y.
- $RECRUIT_{t,y} \geq 0$ New accessions into the force at the E-1 grade during month t of year y.

A number of auxiliary variables are also required in the formulation of the model for mathematical reasons. Principally, they serve to "linearize" the formulation, and thus make the model much easier to solve.

- $\Delta A^{+}_{t,j,y} \geq 0$ Represents the positive part of the difference in month to month advancements. This variable and the following three are introduced to rectify the non-linearity caused by dealing with absolute value terms in the constraints.
- $\Delta A^{-}_{t,j,y} \geq 0$ Represents the negative part of the difference in month to month advancements.
- $\Delta X^{+}_{t,j,y} \geq 0$ Represents the positive part of the difference in month to month strengths.
- $\Delta X^{-}_{t,j,y} \geq 0$ Represents the negative part of the difference in month to month strengths.
- Z Represents the weighted sum of all of the variables in the objective function. This is the value that is minimized during solution of the model.

D. ELASTIC VARIABLES

Elastic variables are the key to goal programming. They allow the model to come as close as possible to its specified goals without actually violating any constraint. The model strives to minimize the weighted values of these variables as it solves for the optimal force structure.

- $G^{+}_{t,j,y} \geq 0$ Positive deviation from EPA target for grade j in month t of year y.

- $G^-_{t,j,y} \geq 0$ Negative deviation from EPA target for grade j in month t of year y.
- $eS_{t,y} \geq 0$ Shortfall from total strength target in month t of year y.
- $eB^-_{j,y} \geq 0$ Shortfall from grade j budget target for year y. Although the budget is not targeted to the individual grade level it was necessary to utilize a j index on this and the following variable in order to scale the objective function into common units of "people".
- $eB^+_{j,y} \geq 0$ Excess over grade j budget target for year y.

E. PENALTY WEIGHTS APPLIED TO ELASTIC VARIABLES

The following values are the weights applied to the various elastic variables in the objective function. During routine operation of the model these are the only values the user would adjust to assess the impact of changing the relative importance of achieving any of the multiple objectives in the strength planning process.

- $W1_j$ = Weight applied to the negative deviation from grade j strength targets.
- $W2_j$ = Weight applied to the positive deviation from grade j strength targets.
- P1 = Penalty applied to personnel cost expenditure shortfalls from budget limit.
- P5 = Penalty applied to personnel cost overexpenditures from budget limit.
- $P2_t$ = Penalty applied to falling short of total strength target in month t. The time index t allows the user to apply a heavier penalty in the later months of the fiscal year when it becomes more critical to meet total strength.
- P3 = Penalty applied to month to month absolute value of the difference in advancements.
- P4 = Penalty applied to month to month absolute value of the difference in strengths.

F. MODEL FORMULATION

1. Constraints

The formulation of the constraints follows from the previous discussion of the various force structure and budgetary considerations that must be explicitly accounted for in the model. The constraints establish the underlying structure of the force and the flows of personnel within the force over time. Additionally, they bound the problem and define the limits within which the model must operate.

a. Budget

$$\sum_{t=1}^{T}\sum_{j=1}^{J}\left((MCOMP_{j,y} * X_{t,j,y}) - eB^{+}{}_{j,y} + eB^{-}{}_{j,y}\right) = BUDGET_{y} \quad , \quad \forall y$$

Ensures that the monthly compensation rate for each grade times the inventory in that grade over all months plus or minus an elastic variable does not exceed the total budget for the fiscal year.

b. Flow Balance Equation

$$\left\{\begin{array}{l} BEGINSTR_{j,t=1,y=1} + RECRUIT_{t,j=1,y} + X_{t-1,j,y} \\ - (LR_{t,j} * X_{t,j,y}) + (GR_{t,j} * X_{t,j,y}) \\ - A_{t,j+1,y,(j\geq 4)} + A_{t,j,y,(j\geq 4)} \\ + (AR_{t,j-1} * X_{t,j-1,y}) - (AR_{t,j} * X_{t,j,y}) \\ + (DR_{t,j+1} * X_{t,j+1,y}) - (DR_{t,j} * X_{t,j,y}) \\ = EPA_{t,j,y} + G^{+}{}_{t,j,y} - G^{-}{}_{t,j,y} \end{array}\right\} , \quad \forall t,j,y$$

Balance monthly flows into and out of each grade accounting for beginning inventory, accessions, gains, losses, advancements, and demotions. G^{+} and G^{-} account for deviations (positive or negative) from the paygrade EPA target for that month, grade, and year. Figure 2 depicts these flows of personnel graphically by showing the Markovian flows as solid arrows and the flows determined by the decision variables as hollow arrows.

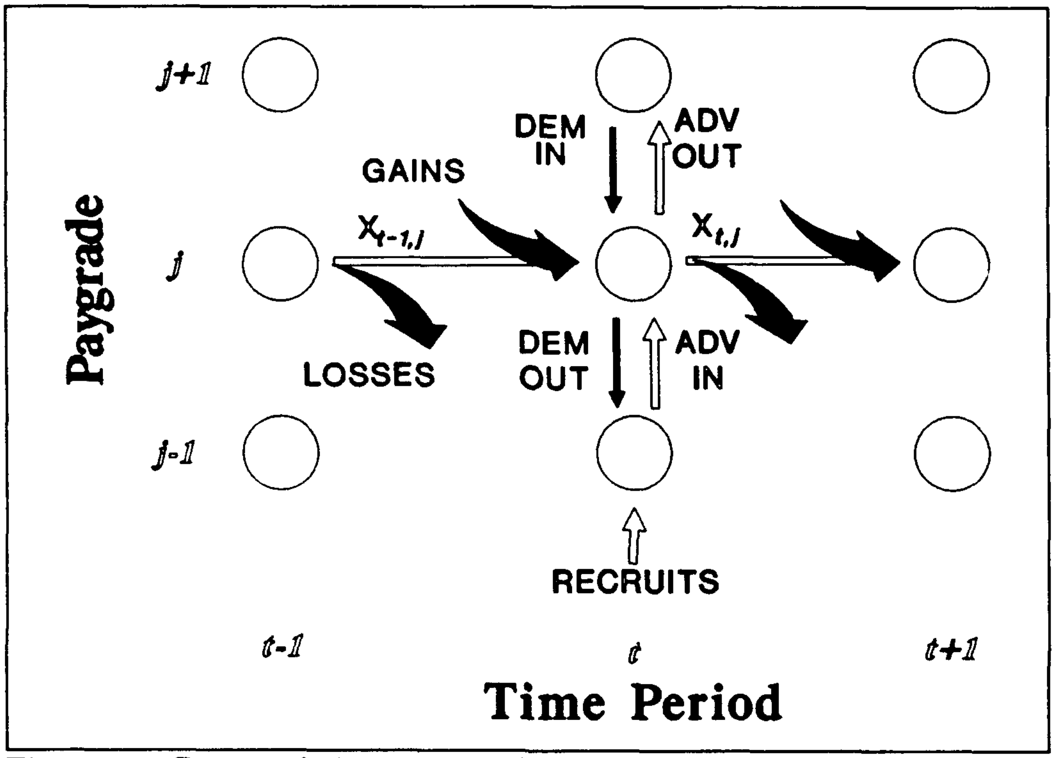

Figure 2. Personnel flows within force structure over time. Solid arcs represent given data as Markovian flows. Hollow arcs represent decision variables.

c. Total Strength

$$\sum_{j=1}^{J} X_{t,j,y} + eS_{t,y} = \sum_{j=1}^{J} EPA_{t,j,y}, \quad \forall t,y$$

Ensures that the total inventory onhand in each month plus an elastic variable does not exceed the total EPA requirement for the month. This constraint also ensures that the Navy will not be "overstrength", i.e., have an inventory that exceeds congressionally mandated ceilings.

d. Inventory Upper & Lower Limits

$$(XLOFACT_j * EPA_{t,j,y}) \leq X_{t,j,y} \leq (XUPFACT_j * EPA_{t,j,y}) \quad ,\forall\ t,j,y$$

Ensures that the inventory in any grade, month, and year remains within some prespecified "bandwidth" of the desired EPA inventory. These bandwidths would be established as a matter of Navy policy.

e. Recruiting Upper and Lower Limits

$$RECRUITLO_t \leq RECRUIT_{t,y} \leq RECRUITUP_t \quad ,\forall\ t,y$$

Ensures that the monthly recruiting goals established by the model remain within limits acceptable to the Navy. At levels lower than $RECRUITLO_t$, the recruit training centers cannot operate effectively due to a lack of manpower. The parameter $RECRUITUP_t$ constrains the recruiting goals to below the maximum capacity of the recruit training centers and accounts for seasonal variations in the ability of the Navy Recruiting Command to bring in new recruits.

f. Advancement Upper & Lower Limits

$$(ALOFACT_j * X_{t,j,y}) \leq A_{t,j,y} \leq (AUPFACT_j * X_{t,j,y}) \quad , \quad \forall t,y,j\geq 4$$

Ensure some minimal upward mobility by advancing at least some percentage of the inventory each month, but limits the maximum number of advancements to a prespecified upper percentage of the inventory. These upper and lower bounds on advancements would be established as a matter of Navy policy.

g. Absolute Difference in Monthly Advancements

$$\Delta A^{+}_{t,j,y} - \Delta A^{-}_{t,j,y} = A_{t,j,y} - A_{t-1,j,y} \quad , \quad \forall t, y, j \geq 4$$

This constraint is required to rectify the nonlinearity caused by dealing with absolute value terms in the difference between monthly advancements. The simplest form of the actual mathematical relationship being modeled is:

$$\textit{Minimize} \ | \ A_t - A_{t-1} \ |$$

h. Absolute Difference in Monthly Strengths

$$\Delta X^{+}_{t,j,y} - \Delta X^{-}_{t,j,y} = X_{t,j,y} - (SLOPE_{t,j,y} * X_{t-1,j,y}) \quad , \quad \forall t,j,y$$

This constraint is analogous to the previous constraint and operates in a nearly identical manner. The SLOPE term is introduced to compensate for inventory requirement changes inherent in the fact that the Navy is declining in strength during the foreseeable future. For example, if the inventory requirement is decreasing by 10 from month t-1 to month t, the model should account for this decrement and not penalize itself for decreasing actual inventories by the same amount.

i. Top 6 Percentage

$$\sum_{j=4}^{J} X_{t,j,y} = T6_y * \sum_{j=1}^{J} X_{t,j,y} \quad , \quad \forall y, t=12$$

Ensures that the percentage of the force in the top 6 grades meets the established Navy goal in the last month of the fiscal year.

j. Top 2 Percentage

$$\sum_{j=8}^{J} X_{t,j,y} \le T2 * \sum_{j=1}^{J} X_{t,j,y} \quad , \quad \forall t,y$$

Ensures that the percentage of the force in the top 2 grades (E-8 & E-9) does not exceed T2% of the total force. In the instance of the model studied, T2 was set at three percent.

k. Top 1 Percentage

$$X_{t,y,j=9} \le T1 * \sum_{j=1}^{J} X_{t,j,y} \quad , \quad \forall t,y$$

Ensures that the percentage of the force in the top grade (E-9) does not exceed T1% of the total force. In the instance of the model studied, T1 was set at one percent.

2. Objective Function Definition

$$Min \sum_{y=1}^{Y}\sum_{t=1}^{T}\sum_{j=1}^{J} \left\{ \begin{array}{c} W1_j * G^-_{tjy} + W2_j * G^+_{tjy} + P1\left(\dfrac{eB^-_{ty}}{12 * MCOMP_{ty}}\right) + P5\left(\dfrac{eB^+_{ty}}{12 * MCOMP_{ty}}\right) \\ + P2_t * eS_{ty} + P3(\Delta A^+_{tjy} + \Delta A^-_{tjy}) + P4(\Delta X^+_{tjy} + \Delta X^-_{tjy}) \end{array} \right\}$$

The objective function contains a summation of each of the multiple objectives the strength planner is concerned with in developing an inventory management plan. It minimizes the following set of objectives:

1. The weighted (by grade) deviation from paygrade EPA targets over all months and years.

2. The weighted shortfall from budget target. The budget terms have been scaled and converted into units of "people".
3. The weighted overexpenditure from budget target.
4. The weighted (by month) shortfall from total strength goals.
5. The weighted difference in month to month advancements. This serves to level advancements over time.
6. The weighted difference in month to month inventories. This serves to level inventories over time.

By simply varying the penalty weights within the objective function, the user can establish relative precedences among the objectives and observe the subsequent outcome of these decisions. This allows for rapid determination of the impact of achieving any individual or combination of goals.

G. MODELING ASSUMPTIONS

Several assumptions about the behavior of the enlisted force, as well as assumptions about the mathematical nature of the model itself were required to make LPSPAN tractable and also allow for solutions in a reasonable amount of time. The most fundamental of these is the treatment of the decision variables in the model as continuous rather than discrete integer values. Obviously people cannot actually be treated as continuous entities (there is no such thing as 2.32 people), however the determination was made that the numbers involved in the model were large enough and there is enough uncertainty about the flows of personnel within the force that using continuous variables and simply truncating the decimals in the output displays was an acceptable approach. Further research into this particular aspect of the model is discussed in Chapter 4.

The assumptions regarding new accessions (recruits) into the force also vary somewhat from actual practice in the Fleet. The model assumes that personnel are only recruited into the E-1 paygrade whereas in the real force personnel are recruited into each of the bottom three paygrades through assorted recruiting incentive programs. Since over 75% of the recruits actually do enter at E-1, the assumption was considered to be reasonable. The distribution of recruits to other paygrades could be accounted for in subsequent versions of the model without much difficulty.

A third assumption concerns the treatment of people as one homogeneous type. The model does not distinguish among various classes and types of personnel to the extent that is actually done in the existing strength planning process. Examples of this include differentiating between male and female recruits, or classifying losses by type (retirement, physical, end of enlistment, etc.).

III. ANALYSIS OF MODEL RESULTS

The analysis presented in this chapter was conducted using a largely graphical approach to assess the cause and effect relationships between the penalty weights in the objective function and the decision variables. To the greatest extent possible, data was used which matched the Navy's in developing plans for fiscal years 1991 through 1993. A set of objective function weights was established as a "baseline" case from which comparisons could be made between existing strength plans developed in the Bureau of Naval Personnel (PERS-22B), and alternative plans developed by LPSPAN. The baseline objective function weights were agreed upon through coordination with Navy strength planners. Table 1 shows the value of the baseline weights that are dimensional in nature. The other weights (P1, P3, and P4) were set at 1, and P5 was set at 100.

TABLE 1. OBJECTIVE FUNCTION WEIGHTS FOR BASELINE CASE

	E-1	E-2	E-3	E-4	E-5	E-6	E-7	E-8	E-9			
W1	1.0	1.25	2.0	2.25	2.5	2.5	2.0	2.0	2.0			
W2	.25	.75	1.5	2.0	2.0	2.0	2.0	2.0	2.0			
	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep
P2	2	4	6	8	10	12	14	16	18	20	22	24

A. INVENTORY COMPARISONS WITH EXISTING PLANS

The total inventories generated by LPSPAN are consistently higher than those generated by the existing strength planning process throughout the three year period encompassed by the instance of the model which was analyzed. Figure 3 shows a comparison of the total inventories created by the two plans.

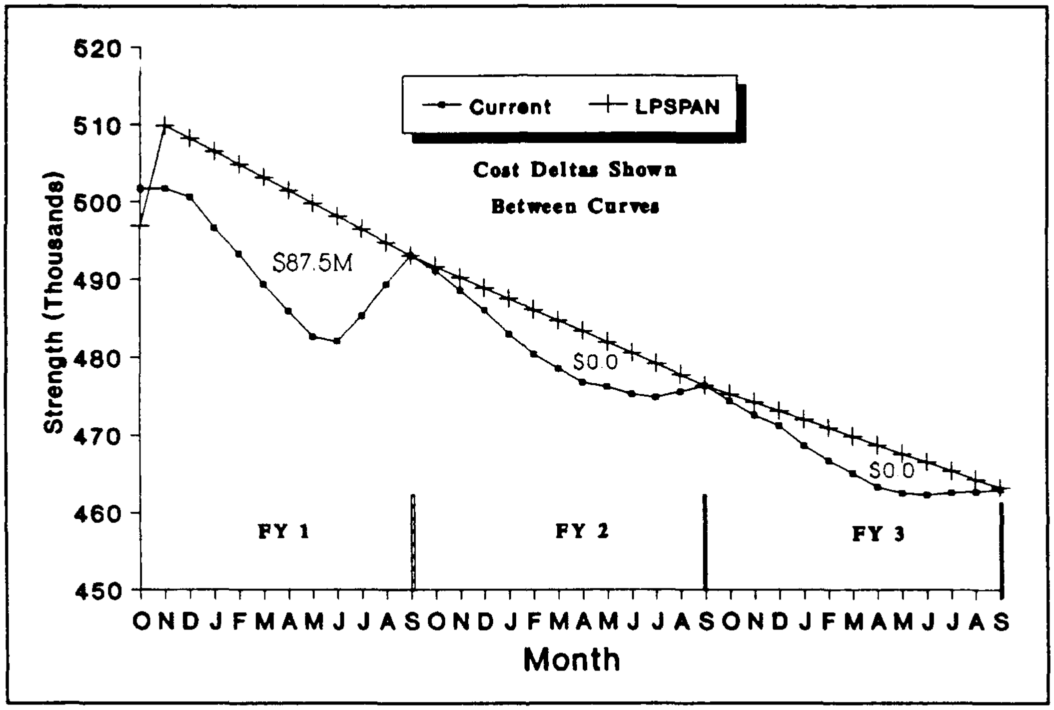

Figure 3. Comparison of Total Strengths.

It should be noted that LPSPAN inventories exactly match the total inventory requirements of the EPA for all but the first month of the first fiscal year, while the existing plans developed by the Navy fall short of these requirements in all but the last month of each fiscal year resulting in insufficient numbers of personnel to man the Fleet most of the time. A comparison of the internal paygrade structures reveals that LPSPAN produces slightly smaller inventories at the higher paygrades but this difference is more than compensated for (+6000) by larger inventories of lower paygrade personnel. These relationships are shown graphically for the middle of the second fiscal year in Figure 4.

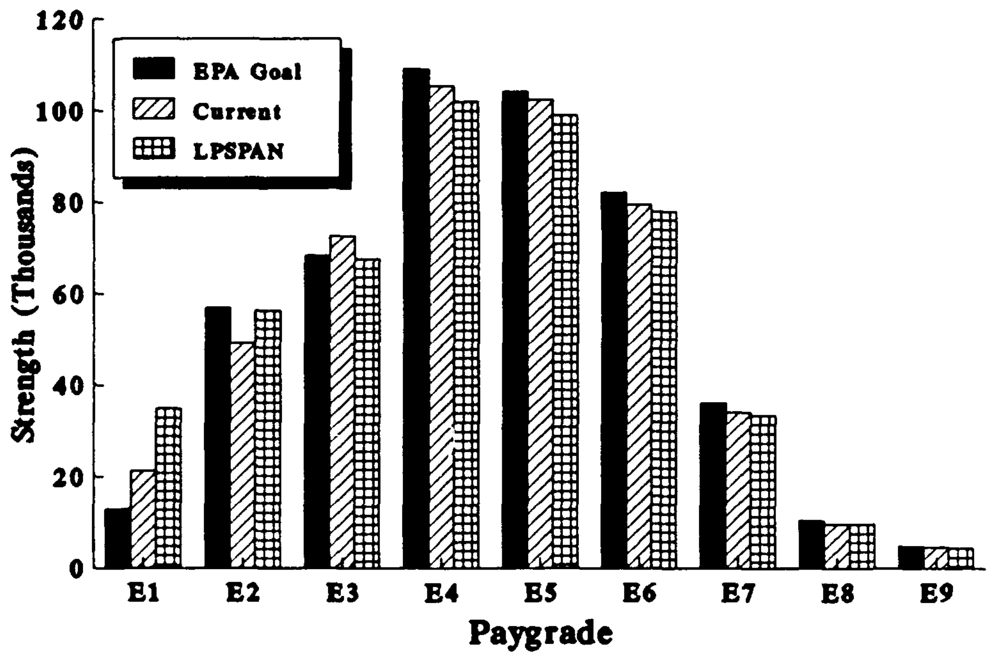

Figure 4. Paygrade Comparison of EPA Goals, Current Plans, and LPSPAN for Mid FY2

Furthermore, LPSPAN creates a more stable paygrade structure over time than the existing plans by avoiding frequent inventory fluctations. Figure 5 shows these effects for the E-5 paygrade as an example. The bounds on inventories shown in the graph apply only to LPSPAN. In this case the bounds are ±5% of the EPA goal. The lower inventory bound has been offset slightly lower for clarity.

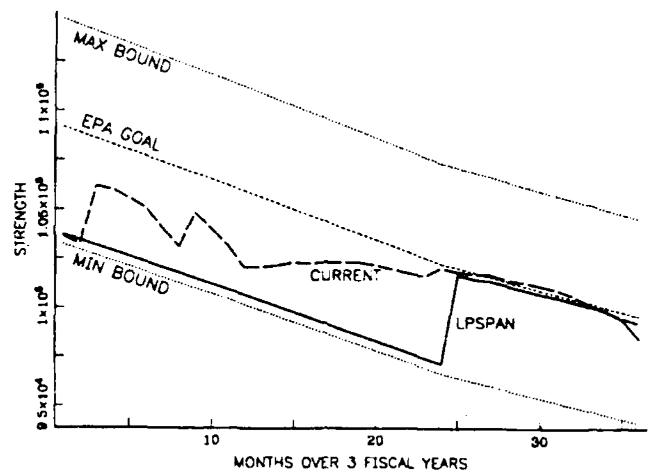

Figure 5. Comparison of Current Plans vs. LPSPAN Showing E-5 Inventories, EPA Goal, and Bounds

Since LPSPAN is constrained to remain within acceptable ranges of each paygrade requirement, the model is able to achieve a larger inventory of personnel to man the Fleet while not seriously impacting the required paygrade structure.

B. BUDGETARY COMPARISONS WITH EXISTING PLANS

The budgetary evaluation portion of LPSPAN is one of its most attractive features when compared to the existing model, which does not provide any detailed cost analysis information. The intention of the model in this area is to provide strength planners with a defensible argument when justifying the cost of the enlisted force. This becomes particularly important in the ongoing competition within the Navy and DoD for limited budget resources. The model was designed to clearly pinpoint and quantify the need for additional funding if required, or in the unlikely event that excess funding has been provided to indicate potential areas of savings. The baseline budget used for comparison was computed by costing the existing strength plans using composite wage rates provided by the Navy. Figure 3 shows (between the curves) the difference in cost between the two plans used for comparison. The $87.5 million difference in Fiscal Year 1 is attributable to the Navy's decision to not even attempt to achieve EPA required levels of inventory throughout the fiscal year. If this had not been the Navy's intention, it could be asserted that an additional budget appropriation of $87.5 million is necessary to adequately man the Fleet. It is unlikely that a reprogramming of budget resources of this magnitude would be politically possible so the model allows the user to reduce the amount of overexpenditures to some acceptable level. Figure 6 shows one possible scenario where only $5.9 million in additional funding is required. It can be seen that total inventories fall short of requirements for the first five months of the fiscal year and but then match requirements for the remainder of the year.

Fiscal years two and three are more interesting cases in that the model is able to achieve larger total inventories (approximately 6000 more) throughout the years at no additional cost to the Navy. This presents a clear illustration of the sub-optimal nature of the current strength planning process and shows the ability of LPSPAN to more efficiently utilize budget resources in developing achievable, affordable plans. Improvements of this magnitude could have a significant positive impact on Fleet manning and readiness in the future.

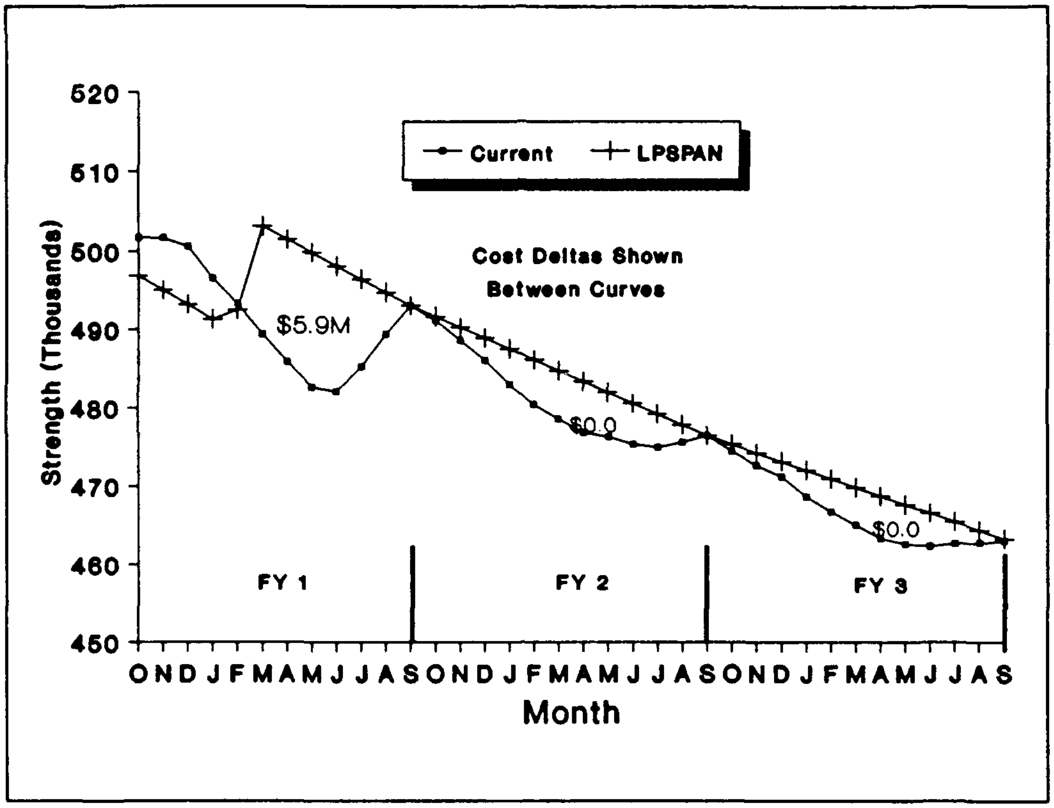

Figure 6. Comparison of Total Strengths with Reduced Cost Differential for FY 1.

C. SENSITIVITY ANALYSIS OF OBJECTIVE FUNCTION WEIGHTS

A sensitivity analysis was conducted for each objective function weight by varying them over a wide range of values to determine the effects on the decision variables. The dimensionless weights (P1, P3, P4, and P5) were varied from .01 to 1000 in an effort to determine ranges of values for which the model seemed to provide the the best possible inventory management plan for the particular instance being studied.

Penalty weight P5 (penalty for budget overexpenditure) seemed to have the most favorable results when set at a value between 50 and 250. Values lower than 50 permitted the model to exceed budget targets by over $250 million in the first fiscal year and over $125 million in the second year. This is still well below what the desired inventory levels of the Navy could cost. If the Navy were to actually achieve the inventories specified in the EPA, the cost would be approximately $300 million per year over the budgets used in analyzing the model. Values above 250 caused the model to remain within the budget ceilings, however inventories in the first year fell significantly short of what is required.

The model proved to be relatively insensitive with respect to advancement stability over the range of values tested for penalty weight P3 (penalty applied to differences in month to month advancements in the top 6 grades). Values as small as .01 produced very stable advancements for the first two fiscal years with some relatively minor turbulence in the third year. Values of 10 or greater produced perfectly stable advancements with no change in any grade over all three years modelled. Based on this, the baseline weight of 1.0 seems reasonable for routine use.

Monthly paygrade inventories for the higher paygrades (E-4 to E-9) remain stable over the full range of values tested for P4 (penalty applied to scaled differences in monthly paygrade strengths). Significant turbulence exists at the lower three paygrades at small values of P4 (.01) which is largely mitigated at values of 1.0 or greater. Values of 50 or greater tend to produce very stable inventories at the paygrade level, however total inventories fall short of requirements. P4 values between 0.5 and 10 seem to produce the best overall performance of the model.

Penalty weight P2 (penalty for falling short of total strength targets) is indexed for each month of the fiscal year to allow the model to place greater emphasis on achieving total strength goals in the latter months of the fiscal year. The penalty value is determined by some multiple of the ordinality of the month of the year (e.g., P2(Jan) = X * 4, since January is the fourth month of the fiscal year). Varying the multiple over a range of values shows that at levels smaller than 0.5 there are shortfalls throughout much of the first fiscal year and a shortfall in the first month of the second year. Multiples below 0.1 tended to produce shortfalls in all of the years modelled, however the model remained within budget ceilings. Based on an overall assessment of the variables involved, it appears that the baseline weight multiple used (2.0) is would be acceptable in most instances.

IV. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

LPSPAN has exhibited the ability to enhance the efficiency of enlisted strength planning in the Navy. Total force inventory improvements of approximately one percent were achieved in the cases analyzed. Although this may appear to be only incremental improvement, in a force of 500,000 personnel with budgets of $13 billion even marginal improvements are significant. The models ability to provide quick, reliable answers to assist in the decision processes involved in strength planning could make it an invaluable aid in the future. The model has been designed to incorporate as many of the objectives and constraints related to the process as possible while retaining the ability to generate solutions in short order on personal computers. These features provide the user the ability to perform rapid "what-if" analysis of the various factors affecting the management of the Navy's most important asset: people.

B. RECOMMENDATIONS

LPSPAN is not intended to supplant the existing enlisted strength planning model in use today, however it is recommended that the model be incorporated into the decision making process as a means of expediting the development and analysis process. The graphical analysis undertaken to explore the relationships among the various objectives and decision variables showed the usefulness of graphics in illuminating the important aspects of the problem. It is recommended that further development of the model be undertaken to incorporate a graphical interface to display the outputs of the model. Well designed graphical displays of the extensive data provided by a model of this type can allow the user to obtain an intuitive understanding of the relationships among the decision variables and penalty weights that cannot be easily ascertained from tabular data.

C. AREAS FOR FURTHER RESEARCH

Further research should be conducted into determination of the personnel flow rates used by the model for those areas where there is no centralized management control. Forecasting the expected loss rates in future years has the most significant impact on the strength plan since the process is essentially designed to replace losses that occur over time. Additionally, the aforementioned graphical interface should also be pursued in order to make the model more user friendly and make the information easier to digest and explain to higher authorities within the Navy. A preliminary exploration has shown the viability of this approach with existing commercial personal computer software which can produce very enlightening graphs (Microsoft Excel 3.0).

Research should also be undertaken to develop an internal algorithm that accomplishes rounding to integer values of the continuous variables in the model [Ref. 10]. Although the method of simply rounding down used in the development of the model is not unreasonable for most practical applications, it could result in some minor ambiguities that could cause confusion to someone unfamiliar with the model.

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